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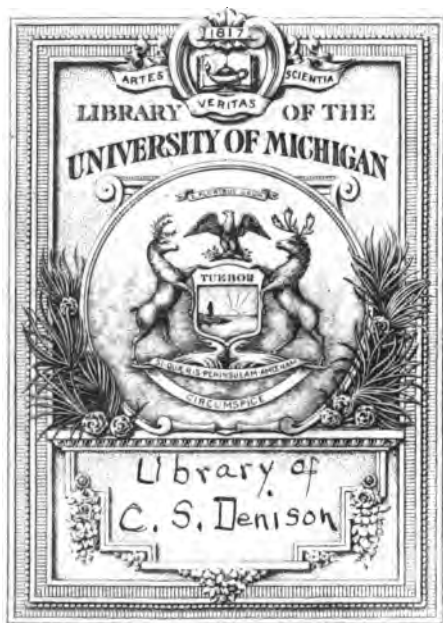
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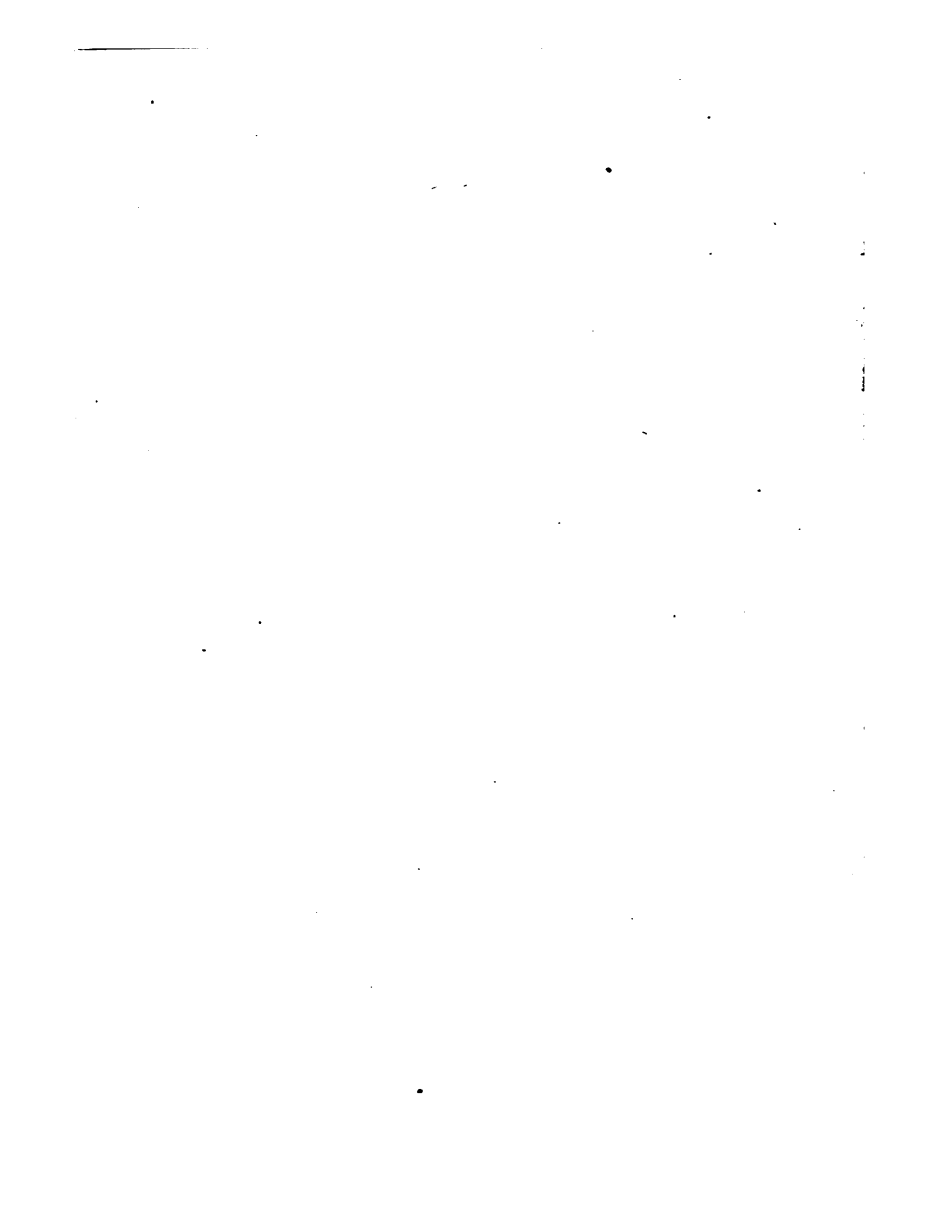


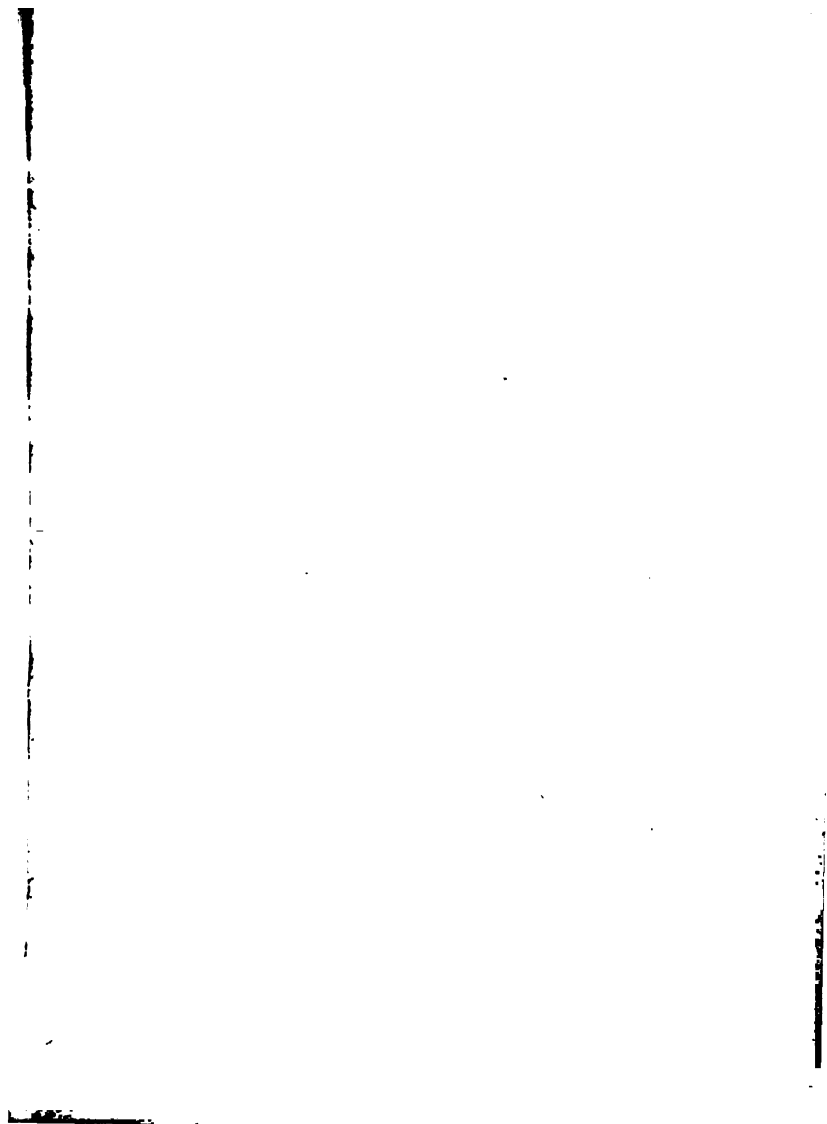
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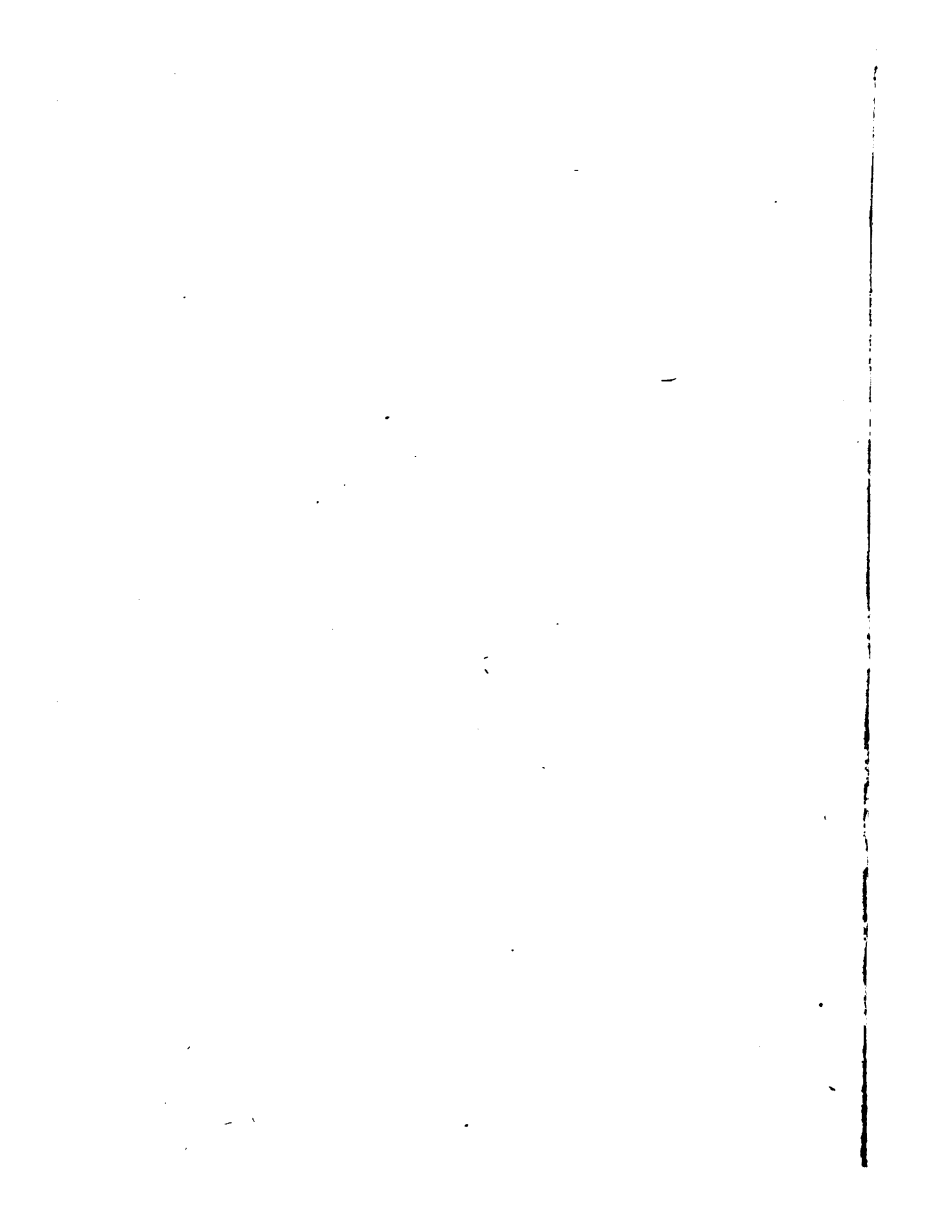
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NOTES
ON
DESCRIPTIVE GEOMETRY
WITH
EXERCISES.

BY

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WORCESTER, MASS.

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PREFACE TO SECOND EDITION.

It is evident to all who have taken note of the trend of the practice in mechanical drawing in the best drafting offices, that the use of the third quadrant in projecting will become universal.

In the study of Descriptive Geometry, however—with few exceptions—the first angle projection is taught. The writer, realizing that the methods taught should harmonize with the practical application in mechanical drawing as practiced, has for some time used the third angle in teaching Descriptive Geometry. There being no text books so arranged, notes were prepared for the students' use, and with such revision as four years' use in the class room would seem to indicate as desirable, these notes are now published.

W. L. A.

PREFACE TO THIRD EDITION.

This edition, though essentially the same as the last, has been revised in the direction of legibility;—obscurities in the text cleared up and new illustrations employed.

In this revision I am indebted to Professor John B. Peddle of the Rose Polytechnic Institute, Terre Haute, Indiana.

W. L. A.

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NOTES ON

DESCRIPTIVE GEOMETRY.

I. Introduction.

Descriptive Geometry treats of methods of representing on plane surfaces, magnitudes of three dimensions in such way that their forms and positions may be completely determined; and conversely, of methods of determining the forms and positions of magnitudes thus represented.

If the intersection, with a plane, of the rays from all points of an object to the observer be taken as the representation of the object on the plane, the production of such a representation requires that three things be known, viz.: the position of the object, of the plane of representation and of the observer.

In Fig. 1, let p represent a point in space and o the position of the observer. A line drawn from one to the other will intersect the vertical plane at p^v , and we say that p^v is the projection of the point p upon this plane. In like manner suppose the

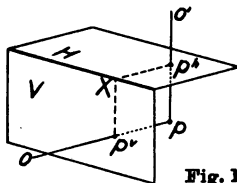


Fig. 1.

position of the observer to be shifted to o' and again connected to p by a straight line.

The intersection of the horizontal plane and this line gives us the point p^h , which is called the projection of p upon the horizontal plane.

Thus knowing the position of the point p in space with reference to the planes of representation, and the position of the observer, or the direction of a line from the point to the observer, we are able to find the projections of the point.

Or, if we reverse the process and have given the point p^v as the projection of the point p upon the vertical plane, when the observer is at o , we know that p must lie somewhere upon the line $o p^v$, though its exact position on the line is indeterminate. Also if p^h is the projection of p upon the horizontal plane, when o' is the observer's position, we know that p lies somewhere upon $o' p^h$. Now having p^v and p^h with the directions $o p^v$ and $o' p^h$, the location of the point p in space is fully determined with reference to the planes of representation.

This method is that used in Descriptive Geometry, but simplified by the following modifications: The planes of representation (which are also used as planes of reference as to position) are taken at

right angles to each other and are considered as unlimited in extent. The points of observation are taken to be at an infinite distance away, so that the rays become orthographic projecting lines and the representations orthographic projections of the object in space.

The two planes (one taken vertical and called the vertical plane of projection or V, the other taken horizontal and called the horizontal plane of projection or H) intersect in a line called the ground line and designated by X. The intersection of the two planes form four

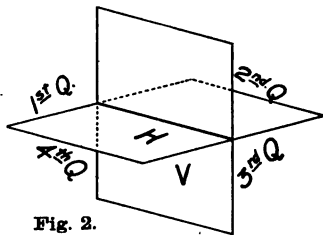


Fig. 2.

dihedral angles called quadrants. The angular space above H and in front of V is called the 1st quadrant, above H and behind V the 2nd, below H and behind V the 3rd, and below H and in front of V the 4th. Fig. 2.

For the convenience of the draftsman the planes are considered as rotated about X, so that the 1st and 3rd quadrants open to 180° and the 2nd and 4th close to 0° .

II. Representation of Points.

In considering the representation of a point as modified by the above conditions, X is taken as a horizontal line, and indicates the line of intersection of H and V , or axis about which H and V are rotated into one plane, which is represented by the plane of the paper. So that above X will appear all the representations or projections, which, when the planes are at right angles, would be in V above H or in H back of V . Hence it will be seen that if a point be in the 1st quadrant its elevation or V projection will be above and its plan or H projection below X . If in the 2nd, both are above. If in the 3rd, the H above and the V below. If in the 4th, both below.

From Fig. 1 it is seen that since $p^v p$ and $p^h p$ are respectively perpendicular to V and H that the projection of all the points of the line $p^h p$ on V will be the line Xp^v perpendicular to the line of intersection of H and V . Also that the projection of the line $p^v p$ on H will be the line Xp^h also perpendicular to the line of intersection of H and V .

Hence when H and V are revolved into the same plane, p^v and p^h will lie in the same perpendicular to X . Fig. 3. The distance from

p^h to X, measuring the distance from p to V. The distance from p^v to X, measuring the distance from p to H.

Fig. 4 shows the projections or representations of a point in each of the four quadrants.

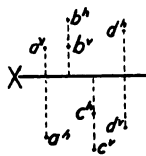


Fig. 4.

From which it is to be noticed that any two

points lying in the same perpendicular to X may be taken as the projections of a point in space.

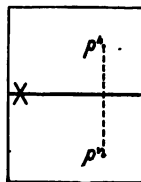


Fig. 3.

NOTATION.—Points in space will be designated by the small letters, as a , b , c . The V projections by the same letters with the exponent v , as a^v , b^v , c^v . The H projections with the same letters with the exponent h , as a^h , b^h , c^h . Successive positions of the same points will be denoted by subscripts, as a^1 , a^2 , a^3 .

PROBLEM 1.—*Having the direction and distance of a point in space, from H and V, to draw its projections.*

Draw any perpendicular to X and set off from X (above if the point be above H, or below if the point be below H) the distance of the point from H; this will be the V projection of the point. On the same perpendicular set off from X (above

X if the point be back of V, or below if the point be in front of V) the distance of the point from V; this will be the H projection of the point.

PROBLEM 2.—*Having one projection of a point in space and the direction and distance of the point from that plane of projection, to draw the other projection.*

A perpendicular to X through the projection given will contain the other projection which will then be located, as in Problem 1.

PROBLEM 3.—*Having the projections of a point in space, to determine its position with reference to H and V.*

The distance of the H projection from X will show the distance of the point from V, and if the H projection be above X the point is back of V; if below X, in front of V. In like manner the V projection of the point will show the distance of the point from H. If above X, the point is above H; if below, the point is below H.

* EXERCISES.

1. Show the projections of the following points:

a, 1" behind V, $1\frac{1}{2}$ " below H.

b, 2" behind V, 1" above H.

c, 3" in front of V, 1" above H.

* NOTE.—In all cases where the conditions of the exercise will permit, the 3rd quadrant is to be used.

d, 1" in front of V, 1" below H.

e, in V, 1" below H.

f, in V, 2" above H.

g, 1" behind V, in H.

k, 1" in front of V, in H.

l, in V, in H.

2. The plan of a point is 1" above X, and the point is $1\frac{1}{4}$ " below H. Show its projections.

3. A point 1" above X is the plan of three points, *a*, *b* and *c*. *a* is 1" above, *b* is in, and *c* is $1\frac{1}{2}$ " below H. Show the V projections.

4. State in which quadrant each of the points shown in Fig. 5 is located, and whether the point is nearer V or H.

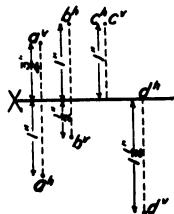
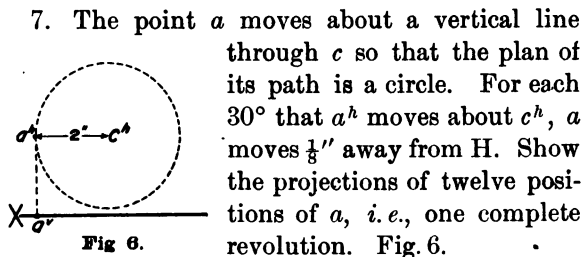


Fig. 5.

5. Three points, *a*, *b* and *c*, lie in a plane perpendicular to X. *a* and *b* have their elevations in the same point, and *b* and *c* have their plans in the same point. *b* is 2" from H and V, *a* is $\frac{1}{2}$ " from V, and *c* is 1" from H. Show the projections of the points.

6. A point in the third quadrant which had its plan $\frac{1}{2}$ " from X and its elevation 2" from X has moved 2" to the right and $\frac{1}{2}$ " down. Show the projections of its former and present positions.



III. Representation of Lines.

The projecting lines of all the points of a right line in space form its projecting plane, the intersection of which with the planes of projection determine the projection of the line in space. Hence, since any two points in the same perpendicular to X may be taken as the projections of a point in space, and since joining the corresponding projections of any two points of a line will be the corresponding projections of the line, it will be seen that any two lines may be taken as the projections of a line in space, provided that they can be cut by two perpendiculars to X .

NOTATION.—Lines in space will be designated by capital letters, as A , B , C . The V projections with the same letter with the exponent v , as A^v , B^v , C^v . The H projections with the same letter

with the exponent h , as A^h, B^h, C^h . A line determined by the point a and b will be called the line \overline{ab} . Also, the point determined by the two intersecting lines C and D will be called the point \overline{CD} .

The points of intersection of a line with the H and V planes of projection are called its H and V traces, and are indicated by h and v .

PROBLEM 4.—*Having the projections of a line, to find its traces.*

The V projection of the H trace of the line must lie in X (since the trace is a point of H) and also in the V projection of the line, since it is a point of the line.

If, therefore, the line $\overline{ab^v}$ be extended till it cuts X and a perpendicular thereto be erected at that point, it must cross the H projection of the line at the H trace. In the same way, the perpendicular to X at its intersection with the H projection of the line will cross the V projection of the line at its V trace. Fig. 7.

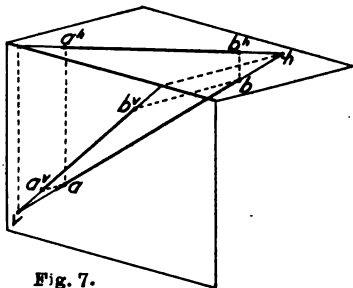


Fig. 7.

the H projection of the line will cross the V projection of the line at its V trace. Fig. 7.

Since the projections of points determine the projections of the lines containing them, exercises in the projection of lines must depend in general on Problems 1 and 2 to determine two points of the line, which is then itself determined.

With reference to H and V a line may have six general positions:

LINE.

- A*, parallel to H, parallel to V.
- B*, parallel to H, perpendicular V.
- C*, parallel to H, inclined to V.
- D*, perpendicular to H, parallel to V.
- E*, inclined to H, parallel to V.
- F*, inclined to H, inclined to V.

H PROJECTION.

V PROJECTION.

- | | |
|--|--|
| <i>A^h</i> , parallel to X. | <i>A^v</i> , parallel to X. |
| <i>B^h</i> , perpendicular to X. | <i>B^v</i> , a point. |
| <i>C^h</i> , inclined to X. | <i>C^v</i> , parallel to X. |
| <i>D^h</i> , a point. | <i>D^v</i> , perpendicular to X. |
| <i>E^h</i> , parallel to X. | <i>E^v</i> , inclined to X. |
| <i>F^h</i> , inclined to X. | <i>F^v</i> , inclined to X. |

Note an exception to *F* when the line is in a plane perpendicular to X. The projections then coincide in a line perpendicular to X and the line in space is indeterminate.

EXERCISES.

8. Represent a line in each of the general positions indicated above.

9. Show the projections and traces of four lines, one crossing each quadrant.

10. Show the projections and traces of the line containing the points a and b (Ex. 1), the projecting lines of the points being 1" apart.

11. Of the line $\overline{c\ d}$ (Ex. 1), projecting lines 2" apart.

12. Show the projections of the triangle having as vertices the points c , d and e (Ex. 1), the projecting lines of c and d coinciding and 1" from the projecting lines of e .

13. Show the projections and traces of a line passing through X and inclined to H and V .

14. Show the projections of a line inclined at 30° to V , parallel to H and 1" from it.

15. The H trace of a line is 2" below X , the V trace 3" to the right and 1" below X . Show the projections of the line.

16. Show the projections of all the triangles that may be formed having as vertices the points a , b , c and d (Ex. 1), the distance of the projecting lines of a to those of b , c and d being respectively 1", 2" and $3\frac{1}{2}$ ".

17. The point \overline{AB} is $1\frac{1}{4}"$ from H and $1\frac{1}{2}"$ from V; the H projection of this point and the H traces of A and B form an equilateral triangle of $1\frac{1}{2}"$ side, of which one side is inclined at 45° to X. Find the V traces of A and B .

18. The point a is at the H trace and the point b at the V trace of the line \overline{ab} . The elevation of the line is inclined at 30° and the plan at 45° to X. The distance $a''b''$ is $2"$. Show the projections of the line.

IV. Representation of Planes.

Planes are generally represented by their lines of intersection with H and V, called H and V

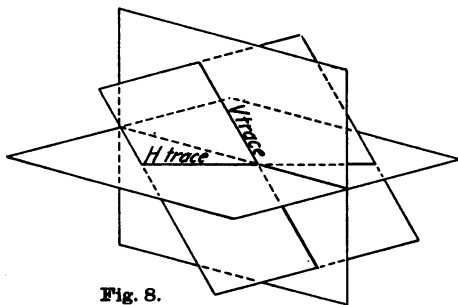
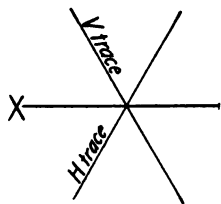


Fig. 8.



traces, forming two lines intersecting in X and extending indefinitely on either side of it. Fig. 8.

For the sake of clearness it is customary to represent only the part of the plane included in one quadrant. The plane shown in Fig. 8 can be represented in four different ways, according as the part of the plane assumed be taken in 1Q, 2Q, 3Q, or 4Q. Fig. 9.

In the following, unless otherwise stated, planes will be taken in the 3rd quadrant. So that the H trace is represented above and the V trace below X.

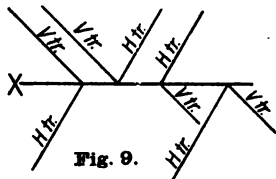


Fig. 9.

NOTATION.—Planes—excepting the planes of projection—will be designated by the numerals, as 1, 2 and 3; the V trace by the same number with the exponent v , as 1^v , 2^v and 3^v ; the H trace by the same number with the exponent h , as 1^h , 2^h and 3^h . The plane determined by the points a , b and c will be called the plane \overline{abc} , and the traces \overline{abc}^h and \overline{abc}^v . The plane determined by two intersecting lines A and B will be called plane \overline{AB} , and the traces \overline{AB}^h and \overline{AB}^v . The plane determined by the line A and the point b will be called the plane \overline{Ab} , and the traces \overline{Ab}^h and \overline{Ab}^v . The line determined by the inter-

section of the planes 1 and 2 will be called line $\overline{12}$, and the projections $\overline{12^h}$ and $\overline{12^v}$. The point determined by the intersection of the planes 1 , 2 and 3 will be called the point $\overline{123}$, and the projections $\overline{123^h}$ and $\overline{123^v}$. The point determined by the plane 1 and the line A will be called the point $\overline{1A}$, and the projections $\overline{1A^h}$ and $\overline{1A^v}$.

There are eight general positions which planes may have have with reference to H and V.

PLANE.

- 1, perpendicular to H, perpendicular to V.
- 2, inclined to H, perpendicular to V.
- 3, perpendicular to H, inclined to V.
- 4, inclined to H, inclined to V.
- 5, perpendicular to H, parallel to V.
- 6, parallel to H, perpendicular to V.
- 7, parallel to but not intersecting X.
- 8, passing through X.

H TRACE.

- 1^h , perpendicular to X.
- 2^h , perpendicular to X.
- 3^h , inclined to X.
- 4^h , inclined to X.
- 5^h , parallel to X.
- 6^h , at ∞ .
- 7^h , parallel to X.
- 8^h , in X.

V TRACE.

- 1^v , perpendicular to X.
- 2^v , inclined to X.
- 3^v , perpendicular to X.
- 4^v , inclined to X.
- 5^v , at ∞ .
- 6^v , parallel to X.
- 7^v , parallel to X.
- 8^v , in X.

EXERCISES.

19. Represent a plane in each of the general positions indicated above.

20. Show the traces of a plane which is perpendicular to V and inclined at 30° to H.

21. Inclined at 45° to V and perpendicular to H.

22. Parallel to V and 1" from it.

23. Parallel to H and $\frac{1}{2}$ " from it.

24. Represent the plane of Ex. 20 in the first quadrant.

25. Show the traces of a plane parallel to X and meeting H and V at an angle of 45° .

V. Representation of Simple Solids.

Solids are represented by showing the form and relative position of the surfaces bounding them. These surfaces are in turn known when we know their determining lines. Hence, solids bounded by plane surfaces are represented by the lines of intersection of these surfaces. Solids bounded by curved surfaces are generally represented by their apparent boundary lines.

EXERCISES.

26. Draw the plan and elevation of a rectangular prism, having the base parallel to H and a

side face parallel to V; the base being 1"x3", and the altitude 2".

27. Show the projections of a triangular prism, axis vertical, and a side face making an angle of 30° with V; base equilateral and of 2" side, altitude 3".

28. Draw the projections of a cylinder, 2" diameter, 2" long, axis perpendicular to V.

29. Show the plan and elevation of a cone of revolution, having for its base a 3" circle in H, while its vertex is 2" below H.

30. Draw the projections of a pyramid, the base being a square of 2" side, situated parallel to V and 3" from it. Vertex is in V.

31. Draw the projections of a 2" sphere whose center is $\frac{1}{2}$ " from H and 2" from V.

32. Fig. 10 shows the plan of a stick, the right section of which is 1" square. Show the elevation.

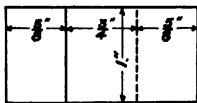


Fig. 10.

VI. Assuming New Planes of Projection.

As has been noted in the representation of lines: When the line is in a plane perpendicular to X, it is indeterminate in direction or po-

sition and may be straight or curved. Hence it is evident that in the representation of objects bounded by plane surfaces, when any of these planes come into a position perpendicular to X , the outline of the surface cannot be determined from the plan and elevation. Therefore it is frequently necessary to show more than two views of a given object. As, for example, a cylinder

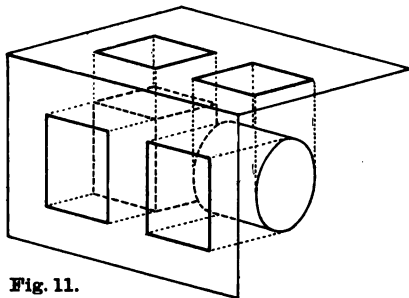


Fig. 11.

having its axis parallel to X will be represented by two equal rectangles, of which the lines parallel to X will represent the sides of the cylinder and the lines perpendicular to X the ends of the cylinder. But these two views would represent as well a prism having an equal breadth and height. Fig. 11. So that to fully determine the form, an end view is necessary.

PROBLEM 5.—*Given the H and V projections of a point in space, to show its projection on a new vertical plane.*

To bring the new vertical plane into the plane of the paper, it is customary to revolve it about its V trace; in which case the H trace of the plane will revolve into coincidence with X , and the projection of the point upon this plane will fall the same distance below X as the V projection. To

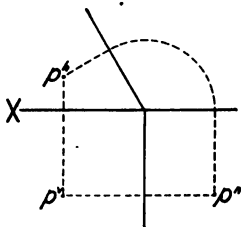


Fig. 12.

find the distance of this new projection to the right or left of the axis of revolution drop a perpendicular from p^h to the H trace of the plane. Fig. 12. The required distance will evidently be the distance between the foot of the perpendicular and the axis of revolution.

The commonest and most important of the planes taken for projection, other than H and V , is the one taken perpendicular to X . This plane is generally called

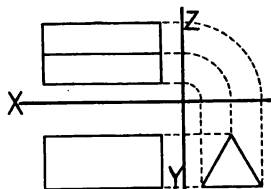


Fig. 13.

the perpendicular plane of projection and will be denoted by P. The line of intersection of P and V will be called Y, and of P and H, Z.

Fig. 13 shows the P projection, or end elevation of a triangular prism.

EXERCISES.

33. Show the P projection of the points a , b and c of Ex. 1.

34. Show the P projection of the line \overline{ab} , Ex. 10.

35. Show the P projection of the triangle cde of Ex. 12.

36. Draw plan and elevation of a bolt $1\frac{1}{8}$ " diameter, 3" under the head; the head being hexagonal, $1\frac{3}{4}$ " short diameter and 1" thick. Take axis parallel to X.

37. Show the projections of a square frame 4" outside, the sides of the frame being 1" square.

NOTE.—Represent hidden lines by short dashed lines, thus — — — —.

38. Show top, side and end view of a bolt having a hemispherical head of 2" diameter. The shank for $1\frac{1}{2}$ " under the head is 1" square, the remaining 2" cylindrical 1" diameter, axis parallel to X.

39. Project a circle of 2" diameter (the plane of the circle being parallel to V), on a plane perpendicular to H and inclined at 60° to V.

40. Project a cube of $1\frac{1}{2}$ " edge on a plane perpendicular to one of its diagonals. [Assume the first position of the cube so that one of its diagonals shall be parallel to H or V.]

41. A cone has for its elevation an equilateral triangle of 2" side. The vertex of the cone is in H and the plane of the base makes an angle of 45° with H. Show plan, front and end elevation.

42. A sphere of $2\frac{1}{2}$ " diameter is pierced by a vertical hole $1\frac{1}{2}$ " square. Show top and front view.

43. A hexagonal prism of 3" length, 2" long diameter, has a cylindrical hole of $\frac{3}{4}$ " diameter half its length. Show plan and elevations with axis taken parallel to X.

44. Show the projections of the box (Fig.14) with the cover raised at an angle of 45° .

45. A rectangular block $1\frac{1}{2}$ " x 2" x 3" has two of its 3" edges in planes $2\frac{1}{4}$ " apart and parallel to H. Show plan and elevation of the block.

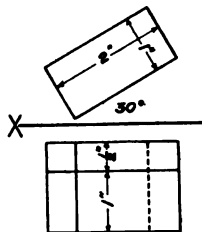


Fig. 14.

46. A set of four equal steps having each a rise and tread of $\frac{1}{2}$ " is 3" long. Show elevation when plan is inclined at 30° to X.

VII. Change of Position by Rotation.

The views obtained by assuming new planes of projection can also be obtained by changing the position of the object with

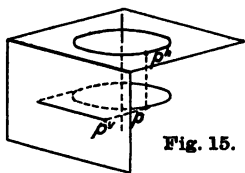


Fig. 15.

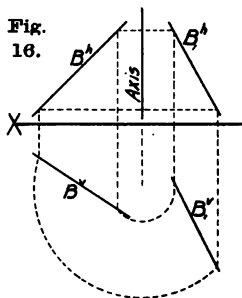
reference to H and V. If, in changing the position by rotation, the axes of rotation be oblique to the planes of projection, the

path of any point in rotating will be projected as ellipses. So that for ease and simplicity of construction it is necessary to take the axes perpendicular to one of the planes of projection. [The result obtained by rotating about an oblique axis can be obtained by two rotations about perpendicular axes—one to V the other to H.] In which case the projection of the path of any given point of the object on the plane to which the axis is perpendicular will be a circle about the intersection of the axis with the plane as a center, while the other projection of the path will be a line parallel to the ground line and

through the corresponding projection of the point. •
Fig. 15.

Fig. 16 shows the rotation of a line about an axis perpendicular to V. Fig. 17 shows a solution of Ex. 40 by rotating the cube so that a diagonal is perpendicular to V.

There are certain cases in which the limitations given above as to the positions of the axes are not necessary. [a] The revolution of a point into either



plane of projection about any line in that plane as an axis.

[b] The revolution of a line into either plane of projection about an axis lying in that plane and passing through the corresponding trace of the line.

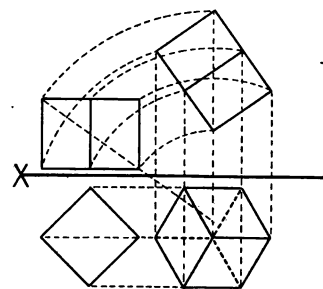


Fig. 17.

[c] The revolution of a plane into either plane of projection about its intersection with that plane as an axis.

PROBLEM 6.—*To revolve a point p into H about an axis in H , containing p^h and inclined to X . Fig. 18.*

Since the plane of revolution of the point will be perpendicular to the axis of revolution, a line drawn through p^h perpendicular to the axis will contain the revolved position of the point. The distance from p to the axis is measured by the distance from p^v to X , so that that distance set off from p^h on the perpendicular will locate the revolved position of p .

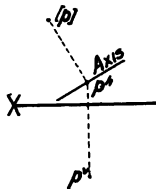


Fig. 18.

The position of a point after revolution into one of the planes of projection will be designated by inclosing the letter in brackets, as $[p]$.

PROBLEM 7.—*To revolve a point into V , about an axis in V , but not containing the V projection of the point. Fig. 19.*

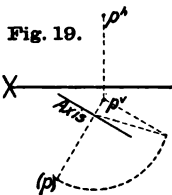


Fig. 19.

In this case, as before, the revolved position of the point will lie on a perpendicular to the axis and passing through p^v . The true distance from the axis to p is the hypotenuse of a triangle having the dis-

tances p^h to X and p^v to the axis as sides. This distance being found by construction is set off from the axis giving $[p]$ as shown.

PROBLEM 8.—*To revolve a line into H about an axis contained in H and passing through the H trace of the line.* Fig. 20.

It is evident that the point of the line which is in common with the axis will not change its position by revolution. So that if any point of the line other than its H trace be assumed and its revolved position found as above, the revolved position of the given line will be the line joining the H trace with the revolved position of the assumed point.

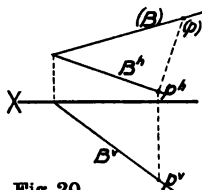


Fig. 20.

PROBLEM 9.—*To revolve a plane about its H trace into H .* Fig. 21.

This result is effected by taking any point of the V trace and revolving it about the H trace as an axis into H . The simplest method of locating $[p]$ is to draw an arc with the center at the intersections of the traces and Sp^v as a radius. Where the arc cuts the perpendicular through

p^h is the point $[p]$, since $S[p]$ and $S p^v$ must be of equal length.

PROBLEM 10.—*To find the true distance between two given points.*

Revolve the line containing the points into, or parallel to one of the planes of projection, whence the corresponding projections of the points will lie at the true distance apart.

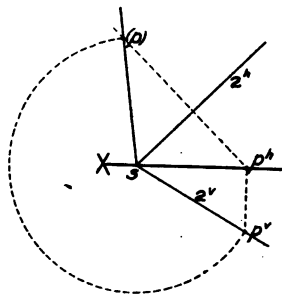


Fig. 21.

EXERCISES.

47. Revolve p through 60° about A as an axis. Fig. 22.

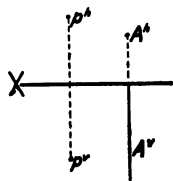


Fig. 22.

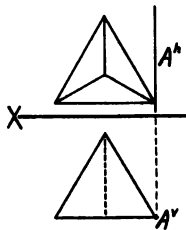


Fig. 23.

48. Revolve the tetrahedron (Fig. 23) through 105° about A .

49. Revolve B into a position parallel to V . Fig. 24.

50. Revolve C into a position parallel to X . Fig. 25.

51. Revolve D into a position perpendicular to V . Fig. 26.

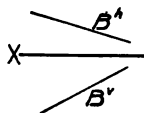


Fig. 24.

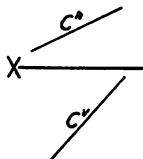


Fig. 25.

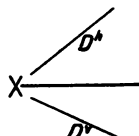


Fig. 26.

52. A square of 2" side stands in a plane perpendicular to X , sides parallel to H and V . Revolve about one of the vertical sides through 60° . Then about an axis perpendicular to V and containing a corner of the square, through 120° .

53. Revolve E into H about E^h as an axis. Fig. 27.

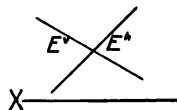


Fig. 27.

54. Revolve F into V about the axis shown in V . Fig. 28.

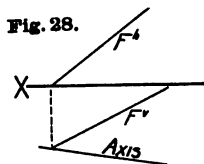


Fig. 28.

55. The P trace of a line is 1" from H and 2" from V . The V trace is $\frac{1}{2}$ " from H and $1\frac{1}{2}$ " from P . Show true distance between H and P traces.

56. The line C whose H projection is given has been revolved into H about the axis, and $[p]$ is a point on its revolved position. Find the V projection of C . Fig. 29.

57. Revolve the plane 2 into H. Fig. 30.

58. Revolve the plane 3 into V. Fig. 31.

59. Revolve the plane 4 into H. Fig. 32.

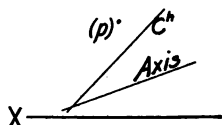


Fig. 29.

60. A line 2" long between its H and P traces, has its H trace 1" from V and $1\frac{1}{4}$ " from P. Its P trace is 1" from H. Show its projections.

Fig. 30.

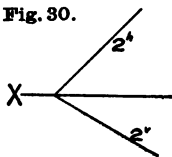


Fig. 31.

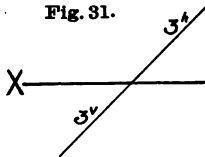
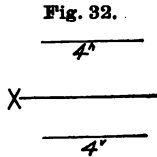


Fig. 32.



61. A rod $2\frac{1}{2}$ " long is suspended horizontally by vertical threads 3" long attached to its ends. Show how far the rod will be raised by turning it through 90° .

62. A rod 2" long stands perpendicular to H at a point 2" from V. A second rod 2" long stands perpendicular to V at a point $1\frac{1}{2}$ " from H. The distance from the foot of one rod to the foot of

the other is 4". What is the distance between the other ends of the rods?

63. The plan of a triangle abc is equilateral of 2" side; ab is $2\frac{1}{2}$ " long, bc is 3". What is the length of ac ?

64. Of the triangle abc , ab is 1", bc is $1\frac{1}{2}$ ", and ac is 2". a and c are in X, b is equi-distant between H and V. Show plan and elevation of the triangle.

VIII. Lines with reference to H and V.

A line in space has the following distinctive data with reference to H and V: H projection, V projection, θ [angle with H] and ϕ [angle with V]. Either two of these are enough to determine the others.

[The H and V traces are important points, but are readily derived from the projections of the line—or the projections from the traces—and in connection with the other data have simply the effect of any other given point of the line.]

PROBLEM 11.—*Given the projections of a line, to find θ and ϕ . Fig. 33.*

[a] To find ϕ revolve the line about its V projection into V, whence the angle between the revolved position of the line and its original V projection is the required angle.

[b] Revolve the line about an axis perpendicular to V till its V projection is parallel to X. The angle between the new H projection and X is the required angle.

The angle θ is found in like manner by using H for V and V for H in the above.

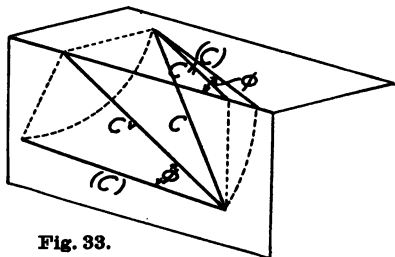


Fig. 33.

PROBLEM 12.—*Given of a line one projection and the angle which the line makes with the corresponding plane of projection, to find the other projection.*

Given A^h and θ . Fig. 34. From any point of A^h draw a line $[A]$, making the required angle with it. This line will represent the line in space revolved parallel to H about A^h as an axis. Making the counter revolution we have the required projection.

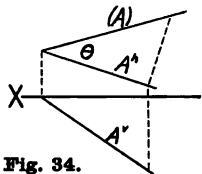


Fig. 34.

Or swing A^h parallel to X, draw the V projection making θ with X, make the counter revolution.

PROBLEM 13.—*Given of a line one projection and the angle which the line makes with the other plane of projection, to find the other projection of the line.*

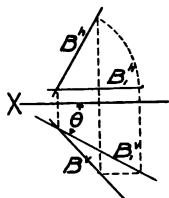


Fig. 35.

Given B^v and θ . Fig. 35. Consider the line revolved into a position parallel to V whence B_i^v will make the angle θ with X and B_i^h will be parallel to X . Making the counter revolution we have the required H projection. [There are two possible positions.]

PROBLEM 14.—*To find the projections of a line making the angle θ with H and the angle Φ with V .*

Suppose b (Fig. 36) to be one of the points of the required line, then all the lines passing through b and making an angle Φ with V will lie on the surface of a cone having b as a vertex and the circle $a''^v a''^v a''^v a''^v$ as a base. The line $b^h a_s^h$ representing the slant height of the cone and inclined at the angle Φ to X . Draw $b^h a_s$ inclined

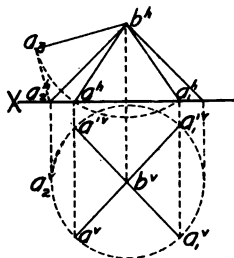


Fig. 36.

at an angle θ with $b^h a_s^h$ and intersect it by a perpendicular to $b^h a_s$ from a_s^h . $b^h a_s$ will then represent the true length of the H projection of the line $a b$. Since this distance must extend from b^h to X, it may be either $b^h a^h$ or $b^h a_i^h$, the consequent V projections being $b^v a^v$, $b^v a_i^v$, $b^v a_j^v$ and $b^v a_k^v$. There being four possible positions of the line.

EXERCISES.

65. Find θ and ϕ for the line $\overline{a b}$. Fig. 37.

66. A line having its H trace $1\frac{1}{2}$ " from X meets V 2" below X. Find θ when ϕ equals 30° .

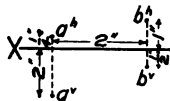


Fig. 37.

67. A pyramid having an altitude of $1\frac{1}{2}$ " has its axis in X and the sides of the base parallel to H and V. The face edges of the pyramid make 30° with H and 45° with V. Show a P projection of the pyramid.

68. A line making 60° with X is the plan of a line which makes 45° with its V projection. Show the V projection.

69. A ray from a point 3" from H and 2" from V is reflected from a point in V $1\frac{1}{2}$ " from H and 2" to the right. Where will the ray meet H?

70. The stick shown incomplete in Fig. 38 has the end face f against H . The other end face is cut to fit against P . The stick is inclined at 30° to V . Complete the views indicated.



Fig. 38.

71. A triangle having sides 4", 3" and 2" lying in H with the 3" side in X , is revolved about the 4" side until the plane of the triangle is vertical. What angle does the 3" side then make with V ?

72. In the orthographic projection of shadows the direction of the projections of the rays of light are assumed as shown in Fig. 39. What angle does a ray make with H and V ?

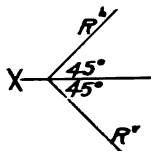


Fig. 39.

73. A ray of light is inclined at 45° to V and its plan is inclined at 60° to X . Assume a point in the first quadrant 1" from V and $\frac{1}{2}$ " from H , and find its shadow on H and V .

74. A line 3" long has one end in H , the other in V . $\theta=15^\circ$, $\phi=45^\circ$. Show the projections of the line.

75. The triangle abc (sides $ab=2''$, $bc=3''$ and $ac=3\frac{1}{2}''$) has the point a in V , $2\frac{1}{2}''$ from H and

$1\frac{1}{2}$ " from P. b is in P, 2" from H. c is in H. Show the projections of the triangle.

76. A cube of 3" edge has a hole drilled at a $\frac{1}{2}$ " deep and at b 1". At what point of the top and at what angle must a drill be started to join the ends of the two holes. Fig. 40.

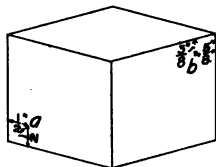


Fig. 40.

77. A sphere which is tangent to both H and V has its center on a line which meets H 2" from X and meets V 4" to the right and 3" from X. Show the diameter of the sphere and the projections of the center.

78. A line drawn through p intersects the lines A and B . Find ϕ and θ . Fig. 41.

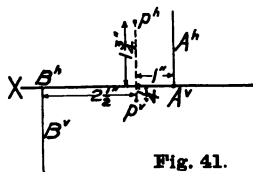


Fig. 41.

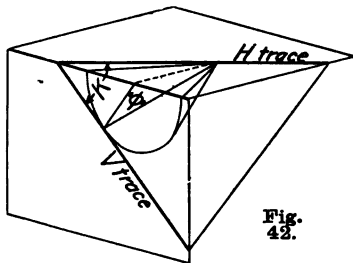


Fig. 42.

IX. Planes with reference to H and V.

A plane in space has the following distinctive data with reference to H and V (Fig. 42): its H

trace, its V trace, the angle K or true angle between the H and V traces, and the angles θ and ϕ , or the angles made by the plane with H and V respectively.

PROBLEM 15.—*Given the traces of a plane, to find the angles of inclination with H and V.* Fig. 43.

If a plane be tangent to a cone of revolution, the plane will make the same angle with the plane of the base that an element does. Hence, to find ϕ construct a cone with base in V and tangent to the V trace of the plane, axis in H and vertex in the H trace of the plane. The angle which the elements of this cone makes with V will be the required angle. To find θ take the base of the cone in H and the axis in V.

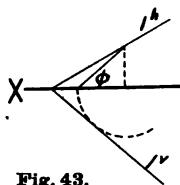


Fig. 43.

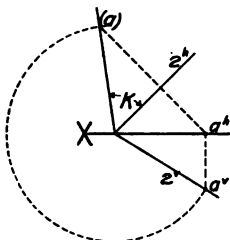


Fig. 44.

PROBLEM 16.—*Given the traces of a plane, to find the angle K between the traces.*

Revolve either trace of the plane about the other as an axis, whence the true triangle will appear. Fig. 44.

PROBLEM 17.—*Given one trace of a plane and the angle which the plane makes with the corresponding plane of projection, to find the other trace.*

Given 3^h and θ . Fig. 45. Assume the base of a cone in H and tangent to 3^h , the axis of the cone being in V. Draw an element of the cone making θ° with H, thus locating the vertex, through which 3^v must pass. There are two solutions of this problem, according as the vertex of the cone is taken above or below X.

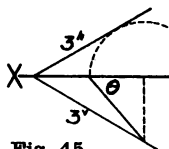


Fig. 45.

PROBLEM 18.—*Of a plane given one trace and the angle which the plane makes with the other plane of projection, to find the other trace.*

Given 4^h and ϕ , to find 4^v . Fig. 46. From any point of 4^h draw a line perpendicular to X, which will represent the axis of a cone having its base in V. From the same point of 4^h draw a line making the angle ϕ with X. This will represent an element of the same cone. Drawing now the base of the cone, 4^v will be tangent to it as shown. There

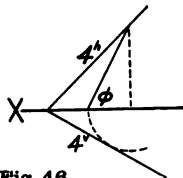


Fig. 46

PROBLEM 21.—*Given the angles θ and ϕ , to draw the traces of the plane.*

Note.—The limits of the value of $\theta + \phi$ are 90° and 180° .

Draw the projections of a sphere having its center in X. Tangent to this sphere draw two cones, one having its axis in V and its elements making an angle θ with H, the other having

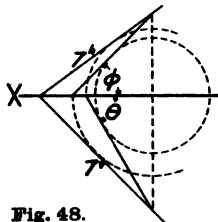


Fig. 48.

its axis in H and its elements making the angle ϕ with V. The required plane is tangent to both cones and will have each trace tangent to the base of one cone and containing the vertex of the other.

EXERCISES.

79. ϑ^h is perpendicular and ϑ^v is inclined at 60° to X. Find θ and ϕ .

80. ϑ^h and ϑ^v appear as one straight line making 60° with X. Find θ and ϕ .

81. ϑ^h is inclined to X at 15° , K equals 120° . Find θ and ϕ .

82. K equals 75° , θ equals 60° . Find the traces of the plane.

83. θ equals 75° , ϕ equals 30° . Find the traces of the plane.

84. A corner of a cube is cut off in such a way that the face left makes 135° with one face of the cube and 120° with the second. What angle does it make with the third face?

85. The H, V and P traces of a plane form a triangle of sides $2''$, $2\frac{1}{2}''$ and $3''$. What angles does the plane make with H, V and P?

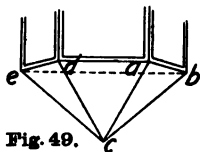
86. β^h is inclined at 60° to Z. $\phi=60^\circ$. Find θ .

87. The oblique section of a square stick makes angles of 75° and 45° with adjacent faces. The longer sides of the section measure $1\frac{1}{2}''$. Show right section of the stick.

88. On a square building having a hip roof the hip rafter makes an angle of 30° with H. What angle does the roof make with H?

89. To what angle must the top of the hip rafter (Ex. 88) be beveled to lie in the plane of the roof?

90. A side face abc of the base of a bay window measures ab , $2\frac{1}{2}'$, ac , $5'$, and bc , $6'$, and is inclined at 45° to H. Show the true form of the mid-



dle face. Fig. 49. Draw to a scale of $\frac{1}{2}''$ to the foot.

91. The edges ac and dc of the base of a bay window are inclined at 30° to the wall. ed and ab measure $2'$, da , $3'$, and eb , $5'$. At what angle is the side face abc inclined to the wall? Fig. 49.

X. Line contained in Plane.

PROBLEM 22.—*Given one projection of a line contained in a plane, to find the other projection.*

Given the plane l and A^h .

Fig. 50. This determines the H trace and H projection of the V trace of A . Since the V projection of the H trace lies in X and the V trace lies in l^v , A^v which contains these points is readily determined.

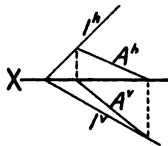


Fig. 50.

PROBLEM 23.—*Given one projection of a point contained in a plane, to find the other projection.*

Through the given projection draw the corresponding projection of a line also contained in the plane, *i. e.*, having its traces in the traces of the given plane. Find the other projection of this line, which will contain the required projection of the point.

PROBLEM 24.—*To find the traces of the plane containing two intersecting lines.*

Since the traces of the line must lie in the cor-

responding traces of the containing plane, the line joining the like traces of the given lines will be the corresponding trace of the required plane.

PROBLEM 25.—*To pass a plane through a given point and a given line.*

Draw a line joining the given point with any point of the given line. The plane of these two lines is the one required.

PROBLEM 26.—*To pass a plane through three given points.*

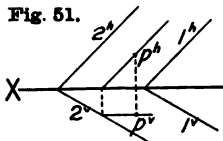
Connect the points by lines so as to form two intersecting lines. The plane of these lines is the one required.

PROBLEM 27.—*To pass a plane through a given point and parallel to two given lines.*

Through the point draw lines parallel to the given lines. Find the plane of these lines.

PROBLEM 28.—*To pass a plane through a given point and parallel to a given plane.* Fig. 51.

Through the given point draw a line parallel to a line of the given plane. [Preferably one of its traces.] The plane containing this line and having its traces parallel to those of the given plane is the one required.



PROBLEM 29.—*To pass a plane through a given line and parallel to a second given line.*

Through any point of the first given line draw a line parallel to the second given line. The plane of these two lines is the one required.

PROBLEM 30.—*To find the lines of a given plane which make a given angle with H or V .*

Take any point of the given plane as the vertex of a cone the elements of which are inclined at the given angle with the plane of the base. The base being in H or V according as the given angle is with H or V . The points of intersection of the circle of the base with the trace of the plane will indicate the elements of the cone which coincide with the plane, and hence are the desired lines.

EXERCISES.

92. Show the traces of the plane \overline{AB} . Fig. 52.

93. The line B is parallel to X , $\frac{1}{2}''$ from H , $1''$ from V . The point a is $\frac{1}{2}''$ from V and $1\frac{1}{2}''$ from H . Find the plane $a\overline{B}$.

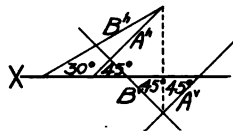


Fig. 52.

94. 2^h makes 30° and 2^v makes 45° with X . A point $\frac{1}{2}''$ from X and $2\frac{1}{2}''$ from 2^v is the elevation

of a point which is $\frac{1}{2}$ " from V. Show the traces of the plane parallel to \mathcal{Q} and containing the point.

95. Show the traces of the plane \overline{abc} . Fig. 53.

96. A rectangle $1'' \times 3''$ (the long side inclined at 30° to X) is the plan of a square the upper edge of which is $\frac{1}{4}''$ below H. Show the elevation of the square and the traces of the plane containing it.

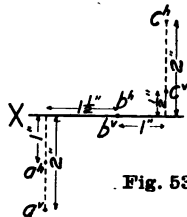


Fig. 53.

97. A triangle in H, sides $1\frac{1}{2}"$, $1"$ and $2"$, has its $2"$ side parallel to X, and is the plan of a triangle situated in a plane which is inclined at 60° with H and 45° with V. Show the elevation of the triangle.

98. A point in plane \mathcal{Q} (Ex. 94) is $\frac{1}{2}$ " from \mathcal{Q}^h and 1" from \mathcal{Q}^v . Show the projections of the point.

99. Find the lines of 4 passing through p and making an angle of 45° with V . Fig. 54.

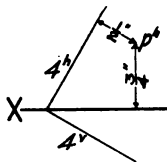


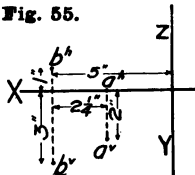
Fig. 54.

100. A point p is $2\frac{1}{2}''$ from H and V. Draw through p two lines each making 60° with H, the plans of which

make 120° with each other. Find the angle between the lines.

101. A ray of light from b is reflected from V (at the point a) to P , thence to H . Find the points of meeting P and H , and the angle at which the ray meets H . Fig. 55

Fig. 55.



102. A regular tetrahedron has edges $2\frac{1}{2}''$ long. Find the line of a face which makes 60° with the plane of the base.

103. The point p lies in the plane def . Find its H projection without using the traces of the containing plane. Fig. 56.

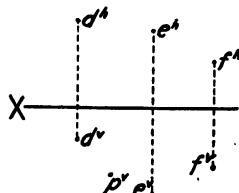


Fig. 56.

104. \mathcal{S}^h is inclined at 30° and \mathcal{S}^v at 45° with X . Show the projections of a line of the plane \mathcal{S} which, when the plane is revolved into H about \mathcal{S}^h will make the same angle with X that it does when revolved into V about \mathcal{S}^v .

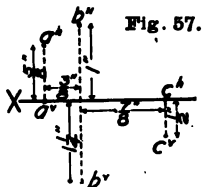


Fig. 57.

105. Find the angle K of the plane abc . Fig. 57.

106. Construct a square on the diagonal $a b$ having its plane vertical. Fig. 58.

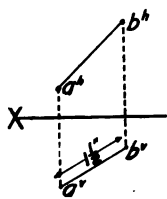


Fig. 58.

107. A and B are the principal axes of the H projection of a circle. Show the principal axes of the V projection of the same circle. Fig. 59.

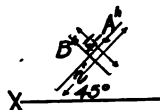


Fig. 59.

XI. Intersections.

PROBLEM 31.—*To find the intersection of two given planes.*

Case a. When the corresponding traces intersect within the limits of the drawing. Fig. 60.

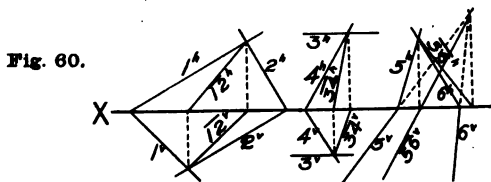


Fig. 60.

The points of intersection of the traces are common to both planes. The line joining these

points will, therefore, be common to both planes, *i. e.*, the line of intersection required.

Case b. When one pair of traces are parallel the others intersecting. Fig. 61. The pair of inter-

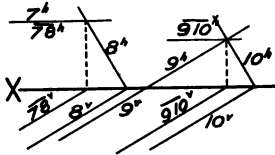


Fig. 61.

secting traces give one point in the required line. The direction of this line will be parallel to the parallel traces; for, if two intersecting planes in-

tersect a third plane in parallel lines, all three intersections will be parallel. (The plane 7 is taken parallel to V.)

Case c. When one pair of traces do not intersect within the limits of the drawing, or when the traces intersect

on X. Fig. 62. A second point in the line of intersection can be found by assuming any auxiliary

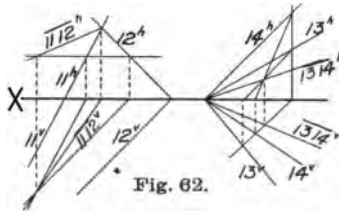


Fig. 62.

plane (preferably perpendicular or parallel to one of the planes of projection) intersecting both the given planes and cutting a line from each which

meet in a point common to all three planes, and hence a point of the required line of intersection.

Case d. When neither pair of traces meet within the limits of the drawing. Fig. 63. In this case two auxiliary planes are taken giving two points in the required line of intersection.

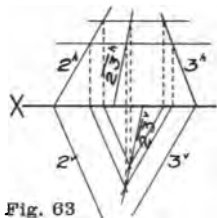


Fig. 63.

PROBLEM 32.—*To find the intersection of a given line and a given plane.* Fig. 64.

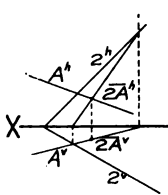


Fig. 64.

Pass any auxiliary plane through the line. [Generally one of the projecting planes of the line will be simplest.] The line of intersection of these two planes will meet the given line in the required point.

EXERCISES.

108. Show the line of intersection of the planes 1 and 2. Fig. 65.

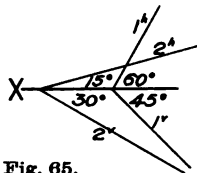


Fig. 65.

109. Show the line of intersection of the planes 3 and 4. Fig. 66.

110. Show the line of intersection of the planes 5 and 6. Fig. 67.

111. Show the point of intersection of the planes 2, 3 and 4. Figs. 65 and 66.

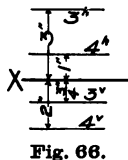


Fig. 66.

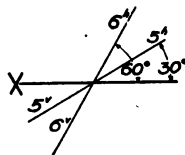


Fig. 67.

112. Consider the traces of the plane 4 as the projections of a line in space, and find its intersection with the plane 2.

113. 6^h is inclined at 45° , and 7^h at 60° to X. θ for 6 is 30° , for 7 is 45° . Find ϕ for the line $\overline{67}$.

114. What length of line parallel to X, 1" from H, 2" from V, is intercepted between the planes 7 and 8? Fig. 68.

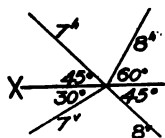


Fig. 68.

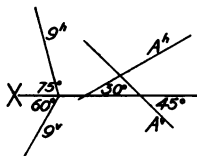


Fig. 69.

115. Show the projections of the point $\overline{9A}$. Fig. 69.

116. The plane 7 (Fig. 68) is intersected by a line whose projections cross at a point $\frac{1}{2}$ " below X and 1" from 7", and are parallel to the corresponding traces of the plane. Find the point of intersection.

117. The H trace of a plane which contains the point a (1" from H, 2" from V) makes 75° , and the V trace 30° with X. A second plane also contains the point a , and has its traces perpendicular to the corresponding traces of the first plane. Find the line of intersection of the two planes.

118. With the point of sight at e show the appearance of the square $abcd$ on the plane P. Fig. 70.

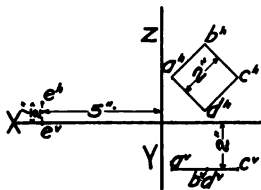


Fig. 70.

119. The base abc of a regular triangular pyramid is in V and of 2" side. The side ab being inclined at 15° to X. Show projections of the vertex d when the face abd is inclined at 30° to H.

120. A shaft A is 5' above and a shaft B is 5' below a floor. The distance between the centers

of the shafts is 16', and they are inclined at 30° to a vertical wall. An 8' pulley on B (clearing the wall by 6") drives a 4' pulley on A (on the other side of the wall) by a 2' belt. Show where the wall and the floor are to be cut for the belt. [Scale, $\frac{1}{4}$ " equals 1'.]

XII. Perpendiculars.

PROBLEM 33.—*Through a given point, to draw a line perpendicular to a given plane.*

If a line be perpendicular to a plane its projections will be perpendicular to the corresponding traces of the plane. Hence, through the given projections of the point draw the projections of the required line perpendicular to the corresponding traces of the given plane.

PROBLEM 34.—*To pass a plane through a given point and perpendicular to a given line.* Fig. 71.

The direction of the traces of the required plane will be perpendicular to the projections of the given line. Hence, through the given point draw a line parallel to one of these traces. The trace of this line will be a point of the corresponding trace of the required plane, which can then be drawn.

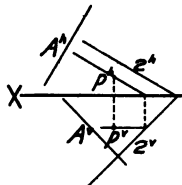


Fig. 71.

PROBLEM 35.—*To pass a plane through a given point and perpendicular to two given planes.*

Through the given point draw perpendiculars to the given planes. The plane of these lines is the one required.

Or, find the line of intersection of the two planes. The required plane is perpendicular to this line.

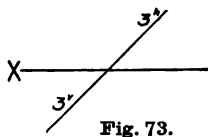
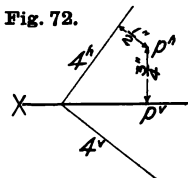
PROBLEM 36.—*Through a given line to pass a plane perpendicular to a given plane.*

From any point of the line draw a line perpendicular to the given plane. The plane of these two lines is the one required.

EXERCISES.

121. Show the point of intersection of the line passing through p and perpendicular to the plane 4 with the plane 4. Fig. 72.

122. A point t in the plane 3 (Fig. 73) is $1''$ from 3^h and $1\frac{1}{2}''$ from 3^v . Show



the H trace of a perpendicular to 3 through t .

123. Three lines, A , B and C , meet at a point o . B makes an angle of 45° with H . A is perpendicular to the plane \overline{CB} . What angle does C make with H ? Fig. 74.

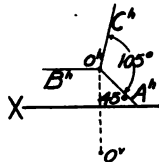


Fig. 74.

124. The H traces of the planes 2 and 3 form with X an equilateral triangle. 2 is inclined at 30° and 3 at 75° with H . Show the traces of a plane perpendicular to 2 and 3.

125. Show the traces of the plane containing A and perpendicular to 2. Fig. 75.

126. 4^v is inclined to X at 60° , 4^h at 45° . Through a point in 4 which is $2''$ from H and $1''$ from V , pass a plane which shall be perpendicular to 4 and also to V .

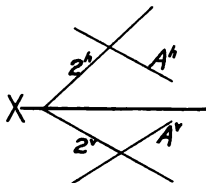


Fig. 75.

127. The H projection of a square having a side in H and an adjacent side in V , is a rectangle having sides of $1\frac{1}{2}''$ and $\frac{3}{4}''$. Project this square on the plane 3. Fig. 73.

128. The H projection of the axis of a square base pyramid is inclined at 30° , and the V pro-

jection at 45° to X . A corner of the base is in H and the adjacent edges of the base are inclined at 30° and 60° to the H trace of the plane of the base. Show the projections of the pyramid.

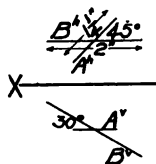


Fig. 76.

129. A and B are the projections of the diagonals of the base of a pyramid of $2''$ altitude. Show its projections. The axis being perpendicular to the plane of the base. Fig. 76.

130. What length of line through p perpendicular to the plane 2 is intercepted between the planes 2 and 3 ? Fig. 77.

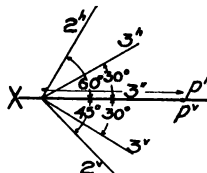


Fig. 77.

XIII. Angles.

PROBLEM 37.—*To find the angle between two intersecting lines.*

Find the plane of the two lines and revolve it with the lines into, or parallel to; H or V , whence the angle will appear in its true size.

Or, draw a line intersecting the given lines, forming a triangle. Find the length of each side and construct the triangle.

PROBLEM 38.—*To find the angle between two given planes.*

Pass a plane perpendicular to the line of intersection. Find the lines cut from the given planes. The angle of these lines is the one required.

Or, from any point in space draw a line perpendicular to each of the given planes. The supplement of the angle of these lines is the one required.

PROBLEM 39.—*To find the angle between a given line and a given plane.*

From any point of the given line draw a perpendicular to the plane. The angle between the line and the perpendicular is the complement of the required angle.

PROBLEM 40.—*To pass a plane through a given line and making a given angle with H or V .*

Construct a cone having its vertex in the given line and its base in H or V as required, and the elements making the given angle with the plane of the base. The corresponding trace of the required plane will contain the trace of the given line and be tangent to the base of the cone. The other trace of the plane will pass through the other trace of the given line.

EXERCISES.

131. What is the angle between the diagonal of a cube and an edge?

132. θ for 2 is 45° , for 3 it is 60° , for the line 23 it is 30° . Find the angle between the planes.

133. A rectangular block is of dimensions $1\frac{1}{2}'' \times 2'' \times 3''$. What angle does a line drawn from a corner to the center of the block make with the plane determined by the three adjacent corners?

134. Two intersecting lines A and B determine a plane. θ for A is 45° , for B , 30° , and for the plane 60° . Find the angle between the lines.

135. A line passing through the corner of a cube makes 75° with one edge and 60° with another. What angle does it make with the third edge?

136. A pyramid having a triangular base of $2''$ side has dihedral angles between its faces of 75° . What is its altitude?

137. A plane is inclined at 60° to H and V . At what angle is it inclined to X ?

138. The H trace of a plane makes 60° with X , and the plane makes 60° with H . This plane contains two lines, one being parallel to, and the other making 45° with H . Find the angle between the lines.

139. A ray from a is reflected from c to b . Show the traces of the reflecting plane. Fig. 78.

140. Given the point and plane of Ex. 126. Draw through the point a line making 30° with the plane and parallel to H.

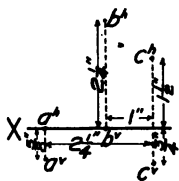


Fig. 78.

141. \varnothing^h is inclined at 45° and \varnothing^v at 30° to X. The line of intersection of the planes \varnothing and \varnothing bisects the angle K for the plane \varnothing . The plane \varnothing is inclined at 30° to H. Show its traces.

142. The traces of a plane make 45° with X. Find the traces of a second plane which is inclined at 60° to H and is perpendicular to the first. Find, also, the line of intersection of the two planes.

143. The planes 2 and 3 are at right angles to each other. 2 is inclined at 60° to H. The line $\overline{23}$ is inclined at 45° to H. What angle does 3 make with H?

XIV. Distances.

PROBLEM 41.—To find the distance from a given point to a given plane.

Through the given point draw a perpendicular to the given plane and find the point of intersec-

tion. Find the distance from this point to the given point as in Problem 10.

PROBLEM 42.—*To find the distance from a given point to a given line.*

Pass a plane through the point perpendicular to the line. Find the point of intersection and distance between points as above.

PROBLEM 43.—*To find the distance between two parallel planes.*

Construct a third plane perpendicular to the given planes. Revolve this with the lines of intersection so as to show their true distance.

Or, draw a line perpendicular to the given planes. Find the point of intersection with each and the distance between these points.

PROBLEM 44.—*To find the shortest distance between two given lines.*

Pass a plane through one of the lines and parallel to the other. Find the length of a perpendicular from any point of the other line to this plane. This is the required distance.

EXERCISES.

144. What is the distance from a corner of a 2" cube to the plane of the three adjacent corners?

145. The H projections of two points are 1" apart. The V projections of the same points are 2" apart. What is the greatest and what the least distance apart that the points can be placed?

146. Show the distance from a to B . Fig. 79.

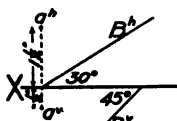


Fig. 79.

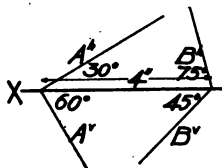


Fig. 80.

147. Show the distance between the lines A and B . Fig. 80.

148. 2^h and 3^h are parallel, $1\frac{1}{2}$ " apart and inclined at 45° to X . The V traces are also parallel and 1" apart. What is the distance between the planes?

149. A line 3" long between its H and V traces, has its H and P traces at the same point. The line makes an angle of 45° with H and 15° with V. Find a point of the P plane which is $2\frac{1}{2}$ " from both ends of the line.

150. A triangular pyramid has a base of 3" side and altitude of 4". Show the projections of

point which shall be respectively $\frac{1}{2}$ ", $\frac{3}{4}$ " and 1" from the three faces of the pyramid.

151. Find the projections of a line parallel to X which is equally distant from the points a , b and c . Fig. 81.

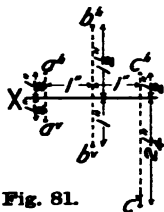


Fig. 81.

152. The line A is inclined at 30° to H and 45° to V. The line B is inclined at 45° to H and 30° to V. The shortest distance between the two lines is 2". Show their projections.

153. A cube of $2\frac{1}{2}$ " edge has its center 2" from H. An edge of the cube has one end $\frac{1}{2}$ " and the other $1\frac{1}{2}$ " from H. Show projections of the cube.

XV. Tangent Planes.

If two lines be tangent to a surface at the same point, the plane of the lines is tangent to the surface at that point. If the surface have straight line elements, the plane will contain the element through the point.

PROBLEM 45.—*Given one projection of a point on the surface of a cone, to pass a plane tangent to the cone and containing the point.*

Suppose the base of the cone in V. Draw the

projections of the element containing the point. The V trace of the required plane is tangent to the base of the cone, and contains the V trace of the element, while the H trace of the plane passes through the H trace of the element.

PROBLEM 46.—*To pass a plane tangent to a given cone, and containing a given point outside its surface.*

Suppose the base of the cone assumed in H. The tangent plane will contain the line drawn through the vertex of the cone and the given point. Hence the H trace of the tangent plane will pass through the H trace of this line and be tangent to the base of the cone. The vertical trace will pass through the V trace of any line drawn from a point of the H trace of the plane through the vertex of the cone.

PROBLEM 47.—*To pass a plane tangent to a given cone and parallel to a given line.*

A line through the vertex of the cone parallel to the given line determines the required plane, as in Prob. 46.

PROBLEM 48.—*To pass a plane tangent to an oblique cone and making a given angle with the plane of the base.*

With the vertex in the vertex of the given

cone construct a cone of revolution, the elements of which make the given angle with the plane of the base. The required plane is tangent to both cones.

Considering the cylinder as a special case of the cone, *i.e.*, with vertex at infinity, the methods of the first three preceding problems will apply as well to the corresponding cases of the cylinder.

PROBLEM 49.—*To pass a plane tangent to an oblique cylinder and making a given angle with the plane of the base.*

Construct a cone of revolution at any convenient place, having its base parallel to the base of the cylinder and its elements making the given angle with the plane of the base. Draw a line through its vertex parallel to an element of the given cylinder. The plane containing this line and tangent to the base of the cone will be parallel to the required plane, which will be found by taking its traces tangent to the corresponding traces of the cylinder and parallel to the traces of the plane found.

PROBLEM 50.—*To pass a plane tangent to a given sphere at a given point on its surface.*

Draw the radius of the sphere to the given

point. The required plane is perpendicular to this line, and contains the given point.

PROBLEM 51.—*To pass a plane through a given line and tangent to a given sphere.*

Pass a plane through the center of the sphere and perpendicular to the given line. This will cut a great circle from the sphere and a point from the given line. From this point draw a tangent to the circle. The plane of this line and the given line is the one required.

Or, from two points on the given line construct cones tangent to the sphere. The intersection of the circles of contact will determine the point of tangency of the required planes. If the axis of one of the tangent cones be taken parallel to H and the other parallel to V, but one ellipse will be required in the construction.

PROBLEM 52.—*To pass a plane tangent to a given surface of revolution and containing a given point on that surface.*

Construct a cone tangent to the surface in a circle passing through the given point. Construct the required plane tangent to this cone at the element passing through the given point.

EXERCISES.

154. A cone of 2" base $1\frac{1}{2}$ " altitude, lies in contact with both H and V. Show the lines of contact.

155. A luminous point 1" above the vertex of a cone is $1\frac{1}{2}$ " from the axis. The cone is of 2" diameter of base and 2" altitude. Find the lines of shade on the cone and the outline of the shadow on the plane of the base.

156. A cone having a vertex angle of 45° has its axis in X. Show the traces of a plane tangent to the cone at a point on its surface $\frac{1}{2}$ " from H and 3" from the vertex.

157. A sphere of 2" diameter, center in H, $\frac{1}{2}$ " from V, is tangent to a plane which is inclined at 45° to H and 60° to V. Show the projections of the point of tangency.

158. A cone which has a vertex angle of 60° has two planes tangent to it at elements which make 30° with each other. What is the angle between the planes?

159. A cylinder has as its base a 2" circle the plane of which is parallel to H. The cylinder is inclined to H at 60° and is parallel to V. Find its shadow on the plane of the base.

160. An anchor ring (axis perpendicular to H) has a tangent plane the H trace of which is inclined at 45° to X. ϕ equals 60° . Find the point of tangency.

161. Find the traces of the plane containing the points a and b and inclined at 75° to H. Fig. 82.

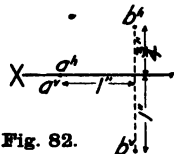


Fig. 82.

162. A sphere 3" in diameter has its center in H, 1" from V. Show the traces of a plane tangent to the sphere at a point 2" from V and $\frac{1}{2}$ " from H.

163. Two spheres of $1\frac{1}{2}$ " and $\frac{3}{4}$ " diameter are tangent to both H and V and each other. Show the traces of the plane tangent to both spheres and having $\theta = \phi$.

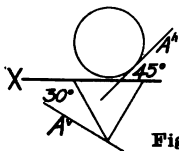


Fig. 83.

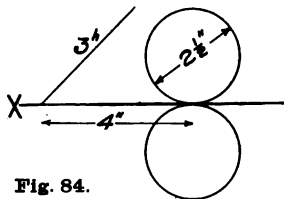


Fig. 84.

164. The line A is the axis of a cylinder which is tangent to the cone. What is the diameter of the cylinder? Fig. 83.

165. Find 3^v if the plane 3 is to be tangent to the sphere. Fig. 84.

XVI. Sections.

When a solid is divided by a plane the surface of division is called a section. If in a prism the cutting plane contains an edge, in a cylinder an element or in a cone the axis, the section is called longitudinal. In a solid of revolution the longitudinal section becomes the meridian section. Since the cutting plane contains the axis and a generating element, all meridian sections are equal. All planes perpendicular to the axis will cut circles called parallels, of which the smallest is called the circle of the gorge, and the largest the circle of the equator.

The general method of finding the section by an oblique plane is to cut both the solid and the plane by auxiliary planes so assumed as to cut the simplest lines (*i. e.*, simplest found) from each. Since the required line of intersection is common to both the secant plane and the surface of the solid, the lines cut from them by an auxiliary plane will intersect in points of the required line.

In order to get simple lines the auxiliary planes should in general be taken as follows:

- (a) In solids bounded by plane surfaces,—coincident with the surfaces or through edges.
- (b) In cylinders,—parallel to the axis.

- (c) In cones,—passing through the vertex.
- (d) In solids of revolution,—perpendicular to the axis.

PROBLEM 53.—*To show the projections and true form of the section of a polyhedron.*

The section is a right lined figure which can be determined by finding the vertices from the intersection of the edges with the cutting plane, or the sides of the figure can be determined by finding the intersection of the planes of the solid with the cutting plane.

The true form of the section can be found by revolving the plane of the section into H or V.

In case the position of the solid can be assumed so as to bring the cutting plane perpendicular to H or V the work is somewhat simplified. One projection being then a straight line coinciding with the oblique trace of the cutting plane. From this the vertices of the other projection can readily be projected.

PROBLEM 54.—*To show the projections and true form of the section of a cylinder of revolution.*

A cylinder of revolution will be cut by a plane in two parallel lines, a circle or an ellipse, according as the cutting plane is parallel, perpendicular or oblique to the axis of the cylinder.

The projections of the section are found by drawing elements of the cylinder and finding their points of intersection with the cutting plane. Enough points should be found to properly determine the outline of the section, which will be a smooth curve joining these points. The true form of the section is found by revolving the cutting plane into H or V as before. If only the form of the section be required and the length of the axes of the ellipse can be determined, any method of constructing the ellipse from its axes may be used.

PROBLEM 55.—*To show the projections and true form of the section of a cone of revolution.*

The section of a cone of revolution (not considering the base) is bounded by two intersecting straight lines if the cutting plane passes through the vertex. Otherwise the section is a circle if the cutting plane be perpendicular to the axis, an ellipse if oblique to the axis but making a greater angle with it than the elements of the cone, a parabola if the same angle as the elements, an hyperbola if a less angle. The method of finding the projections and true form will be as in the case of the cylinder.

PROBLEM 56.—*Show the projections and true form of the section of a solid of revolution.*

Place the solid with the axis perpendicular to H or V. Cut it and the secant plane by auxiliary planes taken perpendicular to the axis of the solid, cutting circles from the solid and lines from the plane which intersect in points of the required line. The true form is found as before.

EXERCISES.

166. A hexagonal prism of 1" side and 3" long has a $\frac{3}{4}$ " hole, the axis of which coincides with the axis of the prism. Show its section by a plane making 60° with the axis.

167. A pentagonal prism, side of base 1", altitude 4", is cut by a plane bisecting the axis and making 45° with it. Show true form of section.

168. Show the true form of the section of a cube by a plane passing through its center and perpendicular to a diagonal.

169. The axis of a cylinder of $1\frac{1}{2}$ " diameter is inclined at 45° to H and 30° to V. The cylinder is tangent to X. Show intersection with H and V.

170. A cone having a vertex angle of 90° has a parabola cut from it by a plane which is $\frac{3}{4}$ "

from the vertex of the cone. Show the true form of the section.

171. A cone of vertex angle of 60° has its vertex $1''$ from H and its axis inclined at 45° to H. Show its intersection with H.

172. Find the profile for a cutter knife which shall cut a moulding of section shown, as-

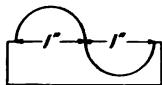


Fig. 85.

suming that the plane of the knife is $1\frac{1}{2}''$ from the axis of rotation, and that the nearest point of the cutting edge is $2''$ from the axis. Fig. 85.

173. A stick of octagonal section (circumscribing circle being $1\frac{1}{2}''$ in diameter) standing parallel to H and perpendicular to V is cut by a vertical plane making 45° with V. Show the right section of a stick standing parallel to V, inclined at 30° to H and exactly meeting the first section.

174. Show the section of a regular tetrahedron, cut by a plane passing through its center and parallel to two adjacent edges.

175. A stick, the section of which is a parallelogram has its V trace and the projection of an edge shown in Fig. 86.

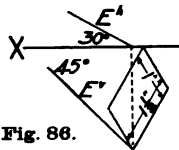


Fig. 86.

What is the angle between the two faces meeting in E ?

176. The stick shown in plan and end elevation is cut by a plane, so that the plan of the section is an equilateral triangle. Show the true form of the section. Fig. 87.



Fig. 87.

XVII. Intersection of Surfaces.

If a given surface made up of planes intersect a second surface the line of intersection will be made up of parts of the sections cut by the planes making up the first surface and may be determined as in the preceding.

In general, to find the line of intersection of two surfaces auxiliary planes are taken cutting lines from the given surfaces which intersect in points of the required line. The amount of work required to find any desired line of intersection will depend largely on the judgment used in selecting auxiliary planes so as to cut simple lines from the given surfaces.

PROBLEM 57.—*To find the projections of the line of intersection of two cylinders.*

Take auxiliary planes parallel to the axes of both cylinders, cutting straight line elements from each, which intersect in points of the re-

quired line. The direction of the traces of these planes can be found by drawing, through any point in space, lines parallel to the axis of each cylinder. The traces of these lines will determine the direction of the traces of the auxiliary planes.

By taking one cylinder perpendicular to V and the other parallel to H the construction will be much simplified.

PROBLEM 58.—*To draw the projections of the line of intersection of a cylinder and cone.*

In this case the cutting planes must pass through the vertex of the cone and be parallel to the elements of the given cylinder. Hence, if a line parallel to the axis of the cylinder be drawn through the vertex of the cone its traces will be common to the like traces of all the cutting planes. Whence they are determined.

PROBLEM 59.—*To draw the projections of the line of intersection of two given cones.*

Since the cutting planes are to pass through the vertices of both cones the line joining the vertices of both cones will be common to all the cutting planes. The traces of which will consequently pass through the traces of this line.

PROBLEM 60.—*To draw the projections of the intersection of a sphere and polyhedron.*

A series of planes parallel to H or V will cut circles from the sphere and polygons from the polyhedron which will intersect in points of the required line of intersection.

PROBLEM 61.—*To draw the projections of the line of intersection of two surfaces of revolution. Axes intersecting.*

Place the surfaces with axes parallel to V and one of the axes perpendicular to H. With center at the intersection of the axes take a series of spheres which intersect the given surfaces in circles, the elevations of which are straight lines and the plan of one a circle and the other an ellipse. [The points of intersection can be determined without use of the ellipse.]

PROBLEM 62.—*To draw the projections of the line of intersection of two surfaces of revolution. Axes not intersecting.*

Place the surfaces as before, one axis perpendicular to H, the other parallel to V. Cut the surfaces by horizontal planes. The elevations of the sections are lines, the plan of one a circle, of the other a curve which will generally have to be

plotted by points, but only a small part of the curve will need to be drawn for each section.

EXERCISES.

177. A is the axis of a $1\frac{3}{4}$ " hole. Show line of intersection. Fig. 88.

178. A pyramid with an altitude of 3" has a base 2" square, parallel to H , the diagonal being parallel to V . It is intersected by a triangular hole of 1" side the axis of which is 1" above the base, $\frac{3}{4}$ " from the axis of the pyramid and inclined at 30° to V . Show plan and elevation of pyramid.

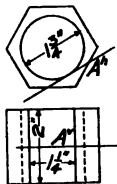


Fig. 88.

179. A point p is $1\frac{1}{2}$ " from a , 2" from b , and $2\frac{1}{2}$ " from c ; a , b and c forming an equilateral triangle in H of $2\frac{1}{2}$ " side. Show the projections of p .

180. Given two spheres, diameters $2\frac{1}{2}$ " and 3", plans of the centers $1\frac{1}{8}$ " apart and on a line making 30° with X . The center of the smaller sphere is $1\frac{1}{4}$ " above H , of the larger in H . Find the traces of the plane, the projections of the center, and the radius of the circle in which the spheres intersect.

181. A vertical tube of $2\frac{1}{2}$ " outside diameter, $1\frac{1}{2}$ " inside, has an horizontal cylindrical hole

bored through it of $1\frac{1}{2}$ " diameter. Distance between axes $\frac{1}{8}$ ". Show projection of line of intersection on a vertical plane to which the axis of the horizontal hole is inclined at 45° .

182. A sphere of 3" diameter is intersected by a cylinder of 2" diameter. The axis of the cylinder is $\frac{1}{2}$ " from the center of the sphere. Show H and V projection of the line of intersection when the axis of the cylinder is parallel to H, inclined at 30° to V and in a horizontal plane with the center of the sphere.

183. A right cone, slant height 3", diameter of base $2\frac{1}{2}$ ", lies against H (the plan of the axis being parallel to X), and is intersected by a cylinder of $1\frac{1}{2}$ " diameter lying against H and inclined at 60° to V. The plan of the axis of the cylinder crosses the plan of the axis of the cone at a point 2" from its vertex. Show projections of the line of intersection.

184. A connecting rod stub-end of rectangular section is finished in a lathe to the outline indicated in Fig. 89. Show complete plan and elevation.

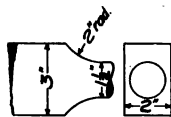


Fig. 89.

185. A surface generated by the revolution of

an arc of 120° about its chord, which is 3" long, is intersected by a cylinder of $1\frac{1}{2}$ " diameter, the axes meeting at the middle point at 45° . Show the projections of the line of intersection.

186. A sphere of $1\frac{1}{2}$ " diameter has its center $\frac{1}{2}$ " above the center of a hemispherical shell of 4" diameter, as shown. Find the shadow of the sphere on the concave surface of the shell. Fig. 90.

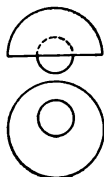


Fig. 90.

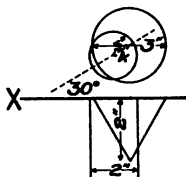


Fig. 91

187. Show the line of intersection of the cone and cylinder of Fig. 91.

188. An anchor ring generated by a 1" circle revolving about an axis $\frac{3}{4}$ " from its center is intersected by a cylinder of 1" diameter, axis of cylinder meeting the axis of the surface at 30° and tangent to the inner surface of the ring. Show projections of the line of intersection.

189. abc is a triangle in H. $ab=3''$, $bc=4''$ and $ac=4\frac{1}{2}''$. d is a point in space. θ for the

line $a d=30^\circ$, for $b d=45^\circ$, for $c d=45^\circ$. Show the projections of d .

XVIII. Development of Surfaces.

Developable surfaces are those which may be brought into complete coincidence with a plane without changing the relative position of any two consecutive straight line elements. Hence, all surfaces having every two consecutive elements in the same plane are developable. The line of shortest distance between two points on a developable surface will, in the developed surface, form a straight line.

PROBLEM 63.—*To develop the surface of a polyhedron.*

The method of development must in general consist in finding the shape and size of each of the faces, and arranging them so that as far as desirable the same edges will be coincident after development as before.

PROBLEM 64.—*To develop the convex surface of a prism.*

Consider the face of the prism as placed against a plane and the prism rolled about each vertical edge—unwrapping its faces as it turns—till each face has come in contact with the plane. The

vertical edges of the prism will remain as parallel lines of distance apart equal to the breadth of the corresponding faces. The total width being the perimeter of the prism. Any right section of the prism will develop into a right line perpendicular to the edges. If the ends of the prism be oblique the lengths of the edges can be set off from any assumed right section.

PROBLEM 65.—*To develop the convex surface of a pyramid.*

If the pyramid be rolled about its edges, as in the case of the prism, the vertex of the pyramid remains at a point and the edges develop as lines radiating from this point. The lengths of the various edges being set off, the remaining boundary line of each face can then be drawn.

PROBLEM 66.—*To develop the convex surface of a cylinder.*

The cylinder being considered as a prism with an infinite number of sides may be developed as the prism. The distance between the first and last element being equal to the perimeter of the cylinder. A right section of the cylinder will develop into a right line perpendicular to the elements. If the cylinder have oblique ends the length of any element may be taken from any

assumed right section and laid off in its developed position from the developed right section line. Enough elements being assumed to determine the developed curve, which can then be drawn through these points.

PROBLEM 67.—*To develop the convex surface of a cone.*

In the case of the oblique cone it may be developed as a pyramid, by assuming points in the base such that the arc and chord between them are practically the same and considering the elements drawn to these points as edges of a pyramid.

A right cone will develop as the sector of a circle having a radius equal to the slant height of the cone, and the arc equal in length to the circumference of the base. The developed angle can be calculated from

$$\frac{\text{Radius of the base}}{\text{Slant height}} = \frac{\text{Developed angle}}{360^\circ}$$

For oblique sections of cones of revolution, elements are assumed, developed position found and true length laid off from the vertex. The curve of the developed base being drawn through the points thus found.

EXERCISES.

190. Show the developed surface of a regular tetrahedron of 2" edge.

191. A pentagonal prism of 1" face is terminated at one end by a plane perpendicular to the axis, at the other end by a plane making 60° with the axis so that the two shortest edges are $1\frac{1}{4}$ " long. Show the development of the convex surface of the prism.

192. A pyramid having a hexagonal base of 1" side, has an edge perpendicular to the base and two inches long. Develop the convex surface.

193. A vertical cylinder of 1" diameter is intercepted between two planes, one inclined at 45° the other at 60° to H. The two planes meeting H in the same line 2" from the axis of the cylinder. Show the development of the convex surface of the cylinder.

194. A cone of revolution, vertex angle of 60° , is cut by a plane making 60° with the axis and cutting the axis 2" from the vertex of the cone. Develop the convex surface.

195. A 2" circle in H is the base of a cone of 2" altitude, the plan of the vertex falling $\frac{1}{2}$ " outside the base. Develop the convex surface.

196. A hexagonal nut of 1" side is chamfered (at an angle of 60° to the axis) to an $1\frac{1}{2}$ " circle. Show the development of the convex surface.

197. A cylinder of $1\frac{1}{2}$ " diameter ends in a triangular pyramid of 1" edge. Develop the surface.

198. A cord is drawn tight over a smooth cone (vertex angle 60°). The cord is tangent to the cone at elements which are 180° apart. At what angle are the ends of the cord inclined to the plane of the base?

XIX. Helicoidal Surfaces.

A helix is the curve traced upon a cylinder of revolution by a point having an uniform motion of rotation about the cylinder and at the same time an uniform motion of translation along the elements of the cylinder. The curve therefore

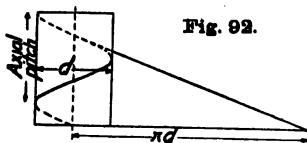


Fig. 92.

makes equal angles with the elements and when the surface is developed forms a straight line which is inclined to the developed elements at

an angle the tangent of which equals the circumference of the cylinder divided by the axial pitch. The axial pitch being the distance between one coil of the helix and the next, measured along an element. Fig. 92.

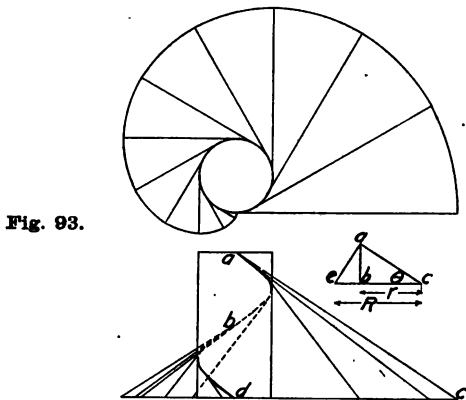
The surface generated by a line moving in contact with a given helix and in some fixed relation to its axis, is called a helicoidal surface. In the ordinary screw surface the generating line intersects the axis; if at right angles the surface formed is a part of the "square" thread, if at 60° the ordinary form of "V" thread.

If a helical surface be intersected by cylinders of revolution having the same axis, the lines of intersection are helices having the same axial pitch but inclined to the base of the cylinder at angles of which the tangent varies inversely as the diameter of the cylinder.

In general, helicoidal surfaces are not developable and are sometimes called "skew screw surfaces."

A single exception occurs in the case when a line moves so as to be in all positions tangent to the given helix, the surface thus generated (since consecutive elements intersect) is developable and is called the developable helicoid.

Fig. 93 represents that portion of the surface included between two parallel planes at a distance apart equal to the pitch of the helix. If a card be cut in the form of the right triangle adc having ad equal to the axial pitch of the helix and dc equal to the circumference of the cylin-



der of the helix, and if this card be placed in the position indicated and rolled about the cylinder in contact with it, the line ac will be in all positions in contact with the helix abd and will therefore generate the desired surface. Two like surfaces will be generated meeting at the helix

in a cuspidal edge. The point a will trace on the upper plane an involute of the plan of the helix, while the point b traces the opposite involute on the lower plane. (Only the lower surface is shown in the figure.)

It can be shown that the cuspidal edge will develop into an arc of radius $R=r \sec^2 \theta$. (r being the radius of the cylinder and θ the angle which an element makes with the plane of the base.) Or, by construction $R=ec$. Fig. 93. The line of intersection of the surface and a plane perpendicular to the axis will develop into an involute of the arc of radius R .

EXERCISES.

199. A helicoidal surface is generated by a horizontal line in contact with a vertical axis and a helix of 2" pitch. Show the projections of the surface included between two horizontal planes 2" apart and two concentric cylinders of 1" and 2" diameter, axis coinciding with the axis of the helix.

200. A "V" thread of 1" pitch, 4" outside diameter, is cut by a plane parallel to the axis and 1" from it. Show the section.

201. A pulley of 2' diameter, 2' 4" above a

floor, is connected by a crossed belt 8" wide to a pulley of 2' diameter and 1' 2" below the floor. The pulleys are 4' apart, center to center. Assuming that the belt in twisting takes the form of a helicoidal surface, show the lines of intersection with the floor. Scale, 1"=1'.

202. A screw conveyor is of the form of a developable helicoid. The diameter of the inner helix is 12", of the outer 30", pitch 36". Show projections and development of one coil. Scale, 1"=1'.

XX. Hyperboloid of Revolution of One Nappe.

This surface may be generated: (1) By the rotation of a line about an axis not in the same plane. (2) By the rotation of an hyperbola about its conjugate axis. (3) By a circle the center of which moves along the conjugate axis of an hyperbola, the plane of the circle being perpendicular to this axis and the circumference of the circle always in contact with the hyperbola.

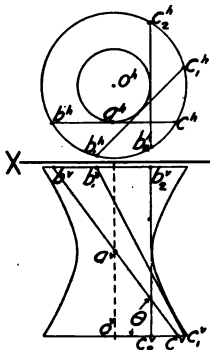


Fig. 94.

The surface is best studied by considering it as generated by the first method.

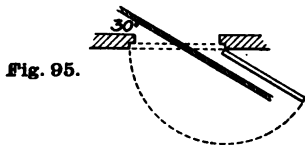
Suppose the line bc , Fig. 94, to rotate about the vertical axis O , keeping at a constant distance Oa from it and making a constant angle θ with H . The line will take the successive positions bc , b_1c_1 , b_2c_2 , etc., being in plan tangent to the circle Oa , from which the elevations are determined.

The surface thus generated will have circles for its cross sections, the smallest being the gorge circle of radius Oa . Any meridian section will be an hyperbola having O as its conjugate axis. It will be seen that the same surface would have been generated if the line having the plan $b^h c^h$ had had the point c in the plane of the top and b in the plane of the base. Hence, the surface is a doubly generated one, being made up of two sets of elements. As a consequence any point on the surface will lie on two straight line elements which are, however, not of the same set. These two elements determine the plane tangent to the surface at that point. From the nature of the generation of the surface no two elements of the same set can intersect or be parallel, hence the surface is not developable.

EXERCISES.

203. Show the projections of an hyperboloid of revolution having the radius of gorge circle 1" and the elements inclined at 30° to the plane of the base. Show also the lines of intersection by planes parallel to V and at distances $\frac{3}{4}$ " and $1\frac{1}{4}$ " from the axis.

204. A rope passes through a doorway as shown in plan in Fig. 95. The door is 3' wide and the rope is inclined at 45° to the horizontal. Show where the door must be cut to allow it to be shut.



205. A surface generated by rotating a cube of 2" edge about a diagonal, is cut by a plane parallel to the axis and 1" from it. Show the outline of the section.

206. A square prism of 1" side is cut by a plane so that the section has angles of 60° and 120° and the long diagonal measures 3". Show the section.